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On conformal deformations

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Abstract: For a conformal theory it is natural to seek the conformal moduli space, $\mathcal{M}_c$ to which it belongs, generated by the exactly marginal deformations. By now we should have the tools to determine $\mathcal{M}_c$ in the presence of enough supersymmetry. Here it is shown that its dimension is determined in terms of a certain index. Moreover, the D-term of the global group is an obstruction for deformation, in presence of a certain amount of preserved supersymmetry. As an example we find that the deformations of the membrane (3d) field theory, under certain conditions, are in $\mathbf{35}/\,\mathrm{SL}(4,\mathbb{C})$. Other properties including the local geometry of $\mathcal{M}_c$ are discussed.

Keywords: Renormalization Group, AdS-CFT and dS-CFT Correspondence, Classical Theories of Gravity, Supergravity Models.

To my daughter, Inbal.

Contents

1. Introduction

Given a field theory one of the most basic properties that one would like to investigate is the vacuum, or in general the moduli space of vacua. Indeed, in the last decade we have learned a lot about moduli spaces of vacua for supersymmetric (susy) field theories. Given a conformal field theory, an equally natural question is to study the moduli space of conformal theories on which it lies, namely the space of parameters for which the theory is exactly conformal. Since both the moduli space of vacua and the conformal moduli space may be interesting for the same field theory I denote the first conventionally by $\mathcal{M}$ and the latter which is the subject of this paper by $\mathcal{M}_c$.

The moduli space of vacua $\mathcal{M}$ is generically expected to be trivial (a point or none) and only in the presence of a certain amount of supersymmetry it is generic to have a non-trivial manifold. Similarly $\mathcal{M}_c$ is expected to be trivial in the absence of any symmetry such as supersymmetry. Therefore our first task, which is the main subject of this paper, is to determine the dimension of $\mathcal{M}_c$.

Looking beyond the dimension we would like to know what is the local geometry of $\mathcal{M}_c$ (once we define its metric) — the relevant analogue of metrics with reduced holonomy. Moreover, we may investigate global issues of $\mathcal{M}_c$ — the location of singularities, (non-)compactness, and possibly a non-trivial topology.

1.1 Summary of results

We begin by discussing some background in section 2. Leigh and Strassler [1] (see also references therein) described a general mechanism to find the dimension of $\mathcal{M}_c$ for 4 dimensional $\mathcal{N}=1$ theories. Despite its success, this formulation raises certain concerns of gauge invariance and scheme dependence. Moreover, it uses cleverly chosen operators with some ad-hoc symmetry properties, breaking the covariance of the problem under the global symmetry, and it is not clear that *all* exactly marginal operators are found in this way.

In order to gain further insight into the issue we use the AdS-CFT correspondence [2] to discuss the analogous concept in gravity, namely the space of vacua which preserves the conformal isometries, or equivalently the AdS factor. In this way $\mathcal{M}_c$ is understood to be related, when a duality exists, to an $\mathcal{M}$ of the (gauged) gravity, a concept that deserves further study, much in same way that the $\mathcal{M}_c$ of the worldsheet theory is the $\mathcal{M}$ of string theory. However, the original formulation of [1] does not translate directly to supergravity. By understanding the supergravity point of view we can abstract properties of $\mathcal{M}_c$ which are independent of the existence of a gravity dual, and refine the formulation of [1] so that the issues raised above are clarified.

In this paper we present the following results

- The conformal index
- A D-term obstruction
- Example: the membrane field theory

which we now discuss one by one.

Like many other deformation problems the (virtual) dimension of $\mathcal{M}_c$ can be formulated in terms of an index (section 3). The relevant operator here is the supersymmetry variation, and in that respect the conformal index can be considered to be a special example of the Witten index [3]. This point of view is certainly standard for 2d field theories, and is hidden in the 4d counting argument of [1], but I am not aware of previous discussions of it in the literature.

An index formulation is useful only after a practical method of computing it is found. Since the index is nothing but the difference of the number of zero modes and the number of obstructions, and moreover the number of zero modes, or supersymmetric marginal deformations, is usually readily determined, we concentrate on the description of the obstruction. In section 4 I show that the D-term for the *global group* is such an obstruction for field theories with 8 super-conformal charges or more (such as 4d $\mathcal{N}=1$). This is not surprising if we remember that the global group becomes the local gauge group in the gravity dual, and in some sense on a general $\mathcal{M}_c$. Moreover, since we may think of the field theory parameters as the VEVs of a background chiral field, we expect $\mathcal{M}_c$ to have a complex structure, and since we wish to divide the space of couplings by the global group, we need to take the D-term constraint in order to achieve a holomorphic quotient. When comparing to [1] the D-term should be considered to be a refinement, or a more precise replacement, for the gamma function constraints used there (the replacement should take place in the NSVZ formula [4] as well).

The case of the field theory of the membrane is discussed in section 5. This is a 2+1 conformal field theory with an SO(8) global symmetry. It depends on a choice of an A-D-E group, which is here taken to be A_N for some large enough N to conform with supergravity. Using the methods described above we find that for deformations that preserve 8 super-conformal charges $\mathcal{M}_c \simeq \mathbf{35}/\,\mathrm{SL}(4,\mathbb{C})$ locally (up to a possible finite N effect). This is a $20_{\mathbb{C}}$ dimensional manifold, where $\mathbf{35}$ is the fourth rank symmetric tensor of SU(4).

1.2 Discussion and open questions

In this paper we discuss the dimension of $\mathcal{M}_c$ and we do not touch much onto the local geometry nor onto global issues. The study of the local geometry can be initiated through the gravity dual by considering the local geometry of $\mathcal{M}$ for 5d gauged supergravity (minimal susy). Quite a lot is known about these theories:[1] the scalars in hypers live in a quaternionic manifold, while the scalars in vectors and tensors live in a "very special geometry", and the metric is related as usual to the kinetic terms of the moduli. The vacuum manifold is determined by $\mathcal{V} = 0$ where $\mathcal{V}$ is the scalar potential. However, the structure of the resulting sub-manifold is not well-understood, but hopefully it is within reach. In addition it would be interesting to determine which field theory couplings types (in 4d $\mathcal{N} = 1$: various superpotential couplings, gauge couplings) correspond to which supergravity multiplets types (in 5d: hyper, vector or tensor).

Hopefully at some point we will have a complete description of some $\mathcal{M}_c$'s (similar to the Seiberg-Witten solution of $\mathcal{M}$ for 4d $\mathcal{N} = 2$), and then we will be able to address some issues about the region of $\mathcal{M}_c$ away from the neighborhood of the origin. It would be interesting to know whether $\mathcal{M}_c$ has any compact factors, and whether there are singularities (we will see that the origin is generally potentially singular because of the quotient structure).

The determination of the dimension of $\mathcal{M}_c$ in this paper is incomplete in some ways. In particular I would like to note here that for theories which may allow susy deformations with fewer than 8 super-conformal charges (such as the membrane theory) a larger $\mathcal{M}_c$ which contains the one found here may exist (currently under study).

A related topic worth pointing out is that spontaneous[2] partial supersymmetry breaking is readily achieved on $\mathcal{M}_c$. For example 4d $\mathcal{N} = 4$ can be deformed to $\mathcal{N} = 1$ (1/4 susy) and similarly for the membrane theory.

2. Background: field theory and gauged supergravity

A d dimensional conformal field theory (CFT) is by definition a Poincaré invariant, scale invariant theory which together with the special conformal transformations is invariant under the full SO$(d,2)$ conformal group.

[1] See [5] for the state-of-the-art.

[2] Spontaneous from the supergravity point of view.

A CFT is conventionally defined as a set of operators together with their correlation functions. Given such a theory, its deformations are one to one with the set of operators, and we may think of deforming with an operator $\mathcal{O}_d$ as adding it to the lagrangian[3]

$$\mathcal{L} \to \mathcal{L} + h\,\mathcal{O}_d \tag{2.1}$$

where h is the expansion parameter. The effect on an arbitrary correlation function

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\rangle = \frac{\int \mathcal{D}\phi\,\mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\ \exp(i\int d^dx\,\mathcal{L})}{\int \mathcal{D}\phi\ \exp(i\int d^dx\,\mathcal{L})} \tag{2.2}$$

(expressed in terms of a functional integral over all fields ϕ) is

$$\begin{aligned}
&\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\rangle \to \\
&\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\rangle_h = \frac{\int \mathcal{D}\phi\,\mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\ \exp[i\int d^dx\,(\mathcal{L}+h\,\mathcal{O}_d)]}{\int \mathcal{D}\phi\ \exp[i\int d^dx\,(\mathcal{L}+h\,\mathcal{O}_d)]} \\
&\qquad = \langle \mathcal{O}_1(x_1),\mathcal{O}_2(x_2)\rangle_0 + h\int d^dx\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\,\mathcal{O}_d(x)\rangle + O(h^2)
\end{aligned} \tag{2.3}$$

and the triple correlation function is understood to be averaged over all orderings. This series is usually referred to as *conformal perturbation theory.*

We would like to study conformal deformations, namely operators $\mathcal{O}_d$ such that the theory remains conformal to all orders of h. To first order this is equivalent to the dimension of $\mathcal{O}_d$ being d, and such operators are called *marginal*, while the ones which are conformal to all orders are called *exactly marginal.* An operator that is both marginal and preserves the supersymmetry of the theory will be called here “super-marginal”.

2.1 Leigh-Strassler

Leigh and Strassler [1] (see also references therein) described how the existence of a non-trivial $\mathcal{M}_c$ may be deduced for some 4d $\mathcal{N}=1$ theories. We shall describe their results here in their language, and later we will see that some refinements are required. Couplings in 4d $\mathcal{N}=1$ may be divided into (complex) gauge couplings and superpotential parameters. A set of couplings h_i is exactly marginal if and only if all their beta functions β_i vanish (to all orders in h_i). The starting point are the exact $\mathcal{N}=1$ formulas for the beta functions in terms of the gamma functions of the charged fields. For the gauge coupling, g, it is the NSVZ formula

$$\beta_g \sim f(g)\,[\beta_0+\gamma] \tag{2.4}$$

in terms of β_0, the 1-loop beta function, and the gamma functions; And for the beta function of a superpotential parameter h ($\delta W = h\,\mathcal{O}$) it is

$$\beta_h \sim h\,[\beta_0+\gamma] \tag{2.5}$$

where $\beta_0 = -\Delta_W + \Delta_{\mathcal{O}}$, $\Delta_W = d-1$ is the dimension of the superpotential in d dimensions, and $\gamma=\gamma(\mathcal{O})$. From this linear dependence it is deduced that for marginal operators ($\beta_0=0$) it is enough to set to zero all the gamma functions. If due to some symmetries

[3] When a lagrangian description is available.

the number of gamma functions is fewer than the number of (super-marginal) couplings, a non-trivial $\mathcal{M}_c$ will exist (generically) simply from counting unknowns and constraint equations.

One concern presents itself immediately, namely, for gauge theories the gamma functions used here are related to 2-point correlation functions $\langle \phi(x_1)\, \phi(x_2) \rangle$ which are not gauge invariant. This can be circumvented either by considering 2-point correlation functions of gauge invariant composites of ϕ such as $\langle \mathrm{tr}(\phi^2)(x_1)\, \mathrm{tr}(\phi^2)(x_2) \rangle$ and trying to read γ off the result, or by defining γ' by inserting a Wilson line into the correlator along some arbitrary line $\langle \phi(x_1) \int_{x1}^{x2} A_\mu(x)\, dx^\mu\, \phi(x_2) \rangle$, and then arguing that at weak coupling γ and γ' become the same, but neither method is quite satisfactory. Another concern raised in the past was the scheme dependence of both the beta functions and the associated gamma functions beyond 1-loop.

2.2 Supergravity

For conformal field theories which possess an AdS gravity dual through the AdS-CFT correspondence [2], let us explore how the issue of $\mathcal{M}_c$ translates into gravity. Suppose the dual is $AdS_{d+1} \times X$, then we would like to deform it keeping all the conformal isometries, which is the same as keeping the AdS_{d+1} factor intact. The most general metric ansatz is an AdS_{d+1} fibered over a deformed X', with some warp function $\rho(x)$, $x \in X$. The ansatz allows to turn on any other fields, with any x dependence, as long as they are AdS scalars.

Since we currently have a working definition of string theory on AdS only in the large radius supergravity limit, we will consider only supergravity fields, and our results translate to the corresponding limits of the field theory, such as 4d field theory in the 't Hooft limit with large 't Hooft coupling, and some 3d theories with a certain large N.

Finding a conformal deformation amounts now to finding a continuous deformation from the zeroth order solution $AdS_{d+1} \times X$ that solves the supergravity equations of motion within this ansatz. Since we are mostly interested in supersymmetric solutions, we will require that the solution satisfies actually the susy variation equations on fermions (which imply the equations of motions). The field mode in the first order (in h) deformation can be directly translated from the supergravity to field theory using the correspondence dictionary. The field theory interpretation of the higher order modes is less clear.

3. The conformal index

Consider finding super-conformal deformations of a gravity solution. In general, the (virtual) number of deformations of a solution to a non-linear set of equations is given by the index of the linearized equations. Let us recall why. One seeks a deformed solution where the fields, denoted here collectively as ϕ, are written as a power series in a perturbation parameters h: $\phi = \sum \phi^{(i)}\, h^i$. After substituting the perturbation series in the equations one attempts to solve the equations order by order. By assumption the zeroth order equations are satisfied, so we go on to the linearized equations, and let us denote that linear operator by L. In a diagonalized form $L\, \phi_j = L_j\, \phi_j$ most field modes ϕ_j will have a non-zero eigenvalue $L_j \neq 0$, however, we take special note of the *zero modes* (or kernel) where

$L_j = 0$, and of the *obstructions* (or cokernel) where the fields do not appear at all at linear orders, namely these modes are outside the image of L. If L has no zero modes then clearly there are no deformations, so we will assume that some do exist. As we go to higher orders the equations will be $L\,\phi^{(k)} = \ldots$ where k is the order and $\ldots$ denotes an expression that depends only on fields from lower orders. For the non-zero-modes there is a unique solution to these equations. For the zero-modes we are free to add terms at any order, but since that amounts only to a redefinition of the perturbation parameters, we choose not to have any such terms beyond the first order. The obstruction equations are the source of trouble — since they are of the form $0 = \ldots$ they are a *constraint* on the first order deformations. In essence the situation is like a set of non-linear equations where the zero modes are the essential unknowns and the obstructions are the essential constraint equations, and so the number of solutions (or the virtual dimension of the solution space) is *generically*

$$\#(\text{zero modes}) - \#(\text{obstructions}) \equiv \text{Index}(L)\,. \tag{3.1}$$

We have to qualify the dimension as "virtual" or "generic" since the actual dimension could be larger if the obstructions are not independent, or it could also be smaller when the equations are not holomorphic.

Considering the index of the supergravity *equations of motion* (e.o.m.) we find that the spaces of fields and equations are the same, so each zero mode is also an obstruction, and hence the index is zero, and generically one cannot expect deformations to exist (this is shown explicitly in the example of [6]). However, as commonly happens, supersymmetry helps. If we consider instead of the equations of motion the *susy variation of fermions*[4] then the fields are bosonic modes while the constraints are fermionic, and hence the zero modes and obstructions are not correlated and the index generically is non-zero. From the point of view of the e.o.m. this shows up as a degeneracy of its obstructions.

Summing up, we see that in supergravity the number of deformations is the index of the linearized susy variations.[5] Finding the explicit index formula for some examples is now work in progress. Proceeding to field theory, since the susy variations in supergravity map to the super-conformal charges, it must be that the number of conformal deformations in field theory is given by their respective index, where this time we include all operators, not only those which are preserved in the supergravity limit. We may call this the *conformal index*, and it is really a special type of the general supersymmetric Witten index.

$$\dim(\mathcal{M}_c) = \text{Index}[\delta_{\text{susy}}] \tag{3.2}$$

The zero modes are the super-marginal (bosonic) operators, for example, in 4d $\mathcal{N} = 1$, consider parameters in the superpotential whose operators have protected dimensions (independent of the gauge couplings), while the obstructions can be identified with certain fermionic operators (or equivalently by their bosonic super-partners), which are less obvious and in the next section I describe some.

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[4] The susy variation of the bosons also play a role in the definition of the index.

[5] I thank E. Witten for an important discussion on this topic.

We can now compare with the language of [1] - their "couplings" must be[6] the zero modes above, and their "gamma function constraints" must be[6] the obstructions. Moreover, in [1] the construction seems to rely on choosing intelligently the operators so they have some high degree of symmetry and break the global symmetry of the problem, whereas the index language does not require that. Nevertheless, formulating the problem in terms of an index is not useful before we find a practical way to compute it. To that purpose we discuss the obstructions in the next section.

4. The D-term obstruction: $\gamma \mapsto D$

In this section we will demonstrate that for CFT's with at least 8 super-conformal charges the D-term of the *global group* is an obstruction to the deformations in the sense of the previous section. Comparing with [1] which identified the obstruction to be gamma functions which have problems with gauge invariance as discussed in section 2, the D-term can be thought to be a refinement summarized by the replacement rule

$$\gamma \mapsto D^I = \sum_h h^\dagger \, T^I \, h \tag{4.1}$$

where the sum is over all scalar fields h, and T^I is a generator of the global group which acts on h according to its representation. This replacement is clearly demonstrated in the following example.

Consider 4d $\mathcal{N} = 4$ with gauge group $\mathrm{SU}(N)$, $N \geq 3$. It has $10_{\mathbb{C}}$ zero modes, h_{ijk} which transform as the third rank symmetric representation of the global SU(3). Leigh-Strassler identified two zero modes operators with special symmetry properties and showed that they are exactly marginal. Alternatively, one can compute the gamma functions for all of the zero modes, and one finds $\gamma_{i\bar{j}} \sim h_{ikl}\, \bar{h}_{\bar{j}\bar{k}\bar{l}}$ where the traceless projection should be taken on both sides (this can be seen at weak coupling from a perturbative evaluation, and is actually always true by [6]). One notices that these constraints are exactly the D-term constraint for the SU(3) representation h_{ijk}, and so together with a division by the global group we get the holomorphic quotient[7]

$$\mathcal{M}_c(4d, \mathcal{N} = 4) \simeq \frac{\mathbf{10}}{\mathrm{SL}(3, \mathbb{C})} \tag{4.2}$$

[6] where equality holds locally near the origin of $\mathcal{M}_c$ (the origin is the $\mathcal{N} = 4$ theory) and higher order corrections are expected away from the origin when additional modes are incorporated.[8] This expression reproduces the same $2_{\mathbb{C}}$ space as [1], only in a more SU(3) covariant way (see [7] and references therein for other recent examples of conformal deformations).

[6] Or at least contain.

[7] Recall that the holomorphic quotient can be defined either by imposing the D-term and dividing by the group, or by dividing by the complexified group, keeping only closed orbits.

[8] The origin of $\mathbf{10}/\,\mathrm{SL}(3, \mathbb{C})$ can be shown to be smooth as a complex variety - namely there are exactly 2 independent gauge invariant coordinates with no relations [6], though the metric may still be singular, exactly like $\mathbb{C}/\mathbb{Z}_k$ which is smooth as a complex variety by working with $z' = z^k$, but may have a conical metric singularity.

With this example in mind, we see why the obstruction had to be the D-term. In 4d $\mathcal{N}=1$ (or other CFT's with 8 super-conformal charges) all parameters may be thought to be VEV's of background chiral multiplets, and as such they should be valued in a space with a complex structure. This property should apply both to the original space of super-marginal operators and to the final space of exactly marginals. Since the transition between the two involves a division by the global group, this division must be done holomorphically, through the use of the D-term.

Supergravity offers another point of view on the D-term. In our example the dual problem is to look for conformal deformations of $AdS^5 \times S^5$ in type IIB. From a 5d $\mathcal{N}=1$[9] point of view type IIB reduced on S^5 is a *gauged* 5d supergravity, with infinitely many KK modes, and with the $\mathcal{N}=4$ multiplets decomposed into $\mathcal{N}=1$ multiplets. The crucial point is that this supergravity has an SU(3) gauge symmetry, and with this amount of supersymmetry vacua are expected to satisfy a D-term constraint (see [5] for the state-of-the-art of this supergravity). Moreover, a detailed analysis of the susy variation equations, shows that a certain phenomenon appears exactly for modes which are associated with complexified gauge transformation [6], as expected from a D-term. In general, one may consider the global group to be a gauge group not only for supergravity duals but also, is some sense, on general $\mathcal{M}_c$'s.

Comparing again to [1] we see that although there is agreement in the examples they studied, the D-term constraint is the more precise definition of the obstruction, it saves us from choosing special symmetric operators breaking the global group and it gives a more unified approach, which allows for generalizations to new results, as we see in the next example.

5. Example: the membrane field theory

As an example for the use of the observations above, let us determine $\mathcal{M}_c$ for the membrane field theory. By "the membrane field theory" I mean the conformal field theory of N coinciding membranes in M-theory (M2 branes). It is a 2+1 dimensional CFT with an SO(8) global symmetry associated with the 8 transverse directions. The theory lacks an intrinsic definition (in particular there is no lagrangian), and one may indirectly define it to be the IR fixed point of a 3d $\mathcal{N}=8$ (maximal) SU(N) gauge theory (hence known as the "A_{N-1} theory"). I would like to show that although we know very little about it, we *can* find the number of exactly marginal deformations.

We approach the problem by considering the gravity dual which is 11d supergravity on $AdS_4 \times S^7$ with a radius that increases with N, so for $N \to \infty$ we may use the 11d supergravity limit. In supergravity we can certainly look for conformal deformations, so we should be able to do it directly in the field theory as well. Actually, from the previous sections we know that all we need to know is the action of the super-conformal charges on the operators of the theory, and whereas little is known about the correlation functions, the spectrum of protected operators *is* known (from supergravity for instance). *Roughly* the

[9] Minimal susy, or 8 supercharges, usually called 5d $\mathcal{N}=2$ in the supergravity literature.

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spectrum is generated[10] by bosonic fields ϕ_i in the $\mathbf{8_v}$ of SO(8) with dimension $\Delta_\phi = 1/2$ (free boson dimension in 3d) and fermionic fields ψ_α in the $\mathbf{8_s}$ of SO(8) with dimension $\Delta_\psi = 1$ (free fermion dimension in 3d).

The first step is to identify the super-marginal operators. We limit ourselves to 3d $\mathcal{N} = 2$ susy deformations in order to be able to use later the D-term constraint. Whether relaxing to $\mathcal{N} = 1$ allows additional deformations is work in progress. 3d $\mathcal{N} = 2$ comes with a $\mathrm{SO}(2)_R = \mathrm{U}(1)_R$ global symmetry and introduces a decomposition of the global group $\mathrm{SO}(8) \to \mathrm{SU}(4) \times \mathrm{U}(1)$. There are two massless scalar representations on $AdS_4 \times S^7$ - a $\mathbf{840_c} = [2,0,2,0]$ of SO(8) from a mode of the 3-form potential and a $\mathbf{1386} = [6,0,0,0]$ of SO(8) from a mode of the metric (mixed with the warp factor).[11] In the field theory they are roughly $\psi^2 \phi^2$ and ϕ^6 [9], respectively. Since the super-marginal modes satisfy a first order differential equation (susy being the "square root" of the second order equations of motion), only the 3-form modes may be super-marginal (the mode $[6,0,0,0]$ is a KK mode for a scalar field and there is no natural first order equation for a scalar). So we proceed to decompose the $\mathbf{840_c}$ according to $\mathrm{SO}(8) \to \mathrm{SU}(4) \times \mathrm{U}(1)$ (embedded such that $\mathbf{8_c} \to \mathbf{6}_0 + \mathbf{1}_2 + \mathbf{1}_{-2}$). Rather than analyze the differential equation we notice that since we want the $\mathrm{U}(1)_R$ to be preserved we need consider only representations with zero R-charge, which are $(\mathbf{35}_\mathbb{C} + \mathbf{84} + \mathbf{45}_\mathbb{C} + \mathbf{20'})_0 = ([4,0,0]_\mathbb{C} + [2,0,2] + [2,1,0]_\mathbb{C} + [0,2,0])_0$ where each complex representation is accompanied by its conjugate. We expect a conformal superpotential to have the form $W \sim h_{ijkl}\, \phi^4$ which in 3d should have dimension 2. Hence I expect that the susy variation equations will select the $\mathbf{35}_0$, the fourth rank symmetric tensor of SU(4) together with its complex conjugate (showing that is work in progress).

The analysis above is valid for $N = \infty$. For finite N we can take 4d $\mathcal{N} = 4$ as a guiding example, and expect that the same operators will be super-marginal for some large enough N (in 4d $\mathcal{N} = 4$ we need $N \geq 3$ for the existence of the d^{ABC} invariant of $\mathrm{SU}(N)$), and in addition other copies of the $\mathbf{35}$ may exist by analogy with multi-trace operators (which were impossible to make out of the ϕ^3 deformation of W in 4d).

Now we may add the D-term to the global SU(4) and conclude that

$$\mathcal{M}_c(3d, \mathcal{N} = 8) \simeq \frac{\mathbf{35}}{\mathrm{SL}(4,\mathbb{C})} \tag{5.1}$$

where as before the equality is local in the neighborhood of the origin (the original membrane theory), $\mathcal{N} = 2$ susy is assumed and the possible finite N effects discussed above should be borne in mind.

One may wonder whether a generalization of (4.2), (5.1) could be interesting, namely $S^N(F)/\,\mathrm{SL}(N,\mathbb{C})$ where $S^N(F)$ stands here for the N^{th} symmetric product of the fundamental representation of $\mathrm{SU}(N)$.

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[10] Though not freely.

[11] See [8], but some shifts in the conventions for m^2 must be performed [9].

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